X 410 “Business Applications of *Calculus*”

PROBLEM SET 1 [100 points]

**PART I**

As manager of a particular product line, you have data available for the past 11 sales periods. This data associates your product line’s units sold “**x**” and total PROFIT “**P**” results for these sales periods.

|  |  |
| --- | --- |
| Product | **Red03** |
| Units [**x**] | Profit [**P**] |
| **10** | **-33986** |
| **20** | **-31792** |
| **100** | **-9200** |
| **130** | **790** |
| **190** | **21418** |
| **240** | **37728** |
| **300** | **54000** |
| **320** | **58208** |
| **380** | **65840** |
| **430** | **65050** |
| **500** | **50000** |

**Section A:** 1st Order Model

1. ***[****4****]*** Use Microsoft Excel’s Chart feature to graph a plot of the data,

assuming **P** = ƒ(**x**). Add the most appropriate 1st order “trend line”, the

equation of this line, and the equation’s *coefficient of determination*—its

“[(*R2*)]”.

2. Answer the following questions using this 1st order model. Assume that, unless otherwise indicated, the restricted domain for “**x**” is **0** ≤ **x** ≤ **510** units.

a. ***[****4****]*** Estimate Profit “**P**” @ “**x**” = **0** units and “**x**” = **70** units.

b. ***[****4****]*** Estimate how many units “**x**” of the product must be sold in order

to generate a PROFIT of $**0.00** and a PROFIT of $**35,000**.

c. ***[****4****]*** Calculate how many product units “**x**” should be sold per sales

period to *optimize* this product’s PROFIT “**P**” and the value of “**P**” at

this “**x**” value. Assume market constraints suggest the maximum

number of product units that actually can be sold per sales period may

not exceed…

(1). …**510** (0 ≤ **x** ≤ 510 units). (2). …**300** (0 ≤ **x** ≤ 300 units).

d. ***[****4****]*** Estimate *marginal* PROFIT “*m***P**” for this product if initially…

(1). …**480** units were sold. (2). …**300** units were sold.

**Section B:** 2nd Order Model

1. ***[****5****]*** Use Microsoft Excel’s Chart feature to graph a plot of the data,

assuming **P** = ƒ(**x**). Add the most appropriate 2nd order “trend line”, the

equation of this line, the equation’s *coefficient of determination*—its

“[(*R2*)]”—and its adjusted *coefficient of determination*—its “[(*R2*)adj]”.

2. Answer the following questions using this 2nd order model. Assume that, unless otherwise indicated, the restricted domain for “**x**” is **0** ≤ **x** ≤ **510** units.

a. ***[****4****]*** Estimate Profit “**P**” @ “**x**” = **0** units and “**x**” = **70** units.

b. ***[****4****]*** Estimate how many units “**x**” of the product must be sold in order

to generate a PROFIT of $**0.00** and sold in a PROFIT of $**35,000**.

c. ***[****4****]*** Calculate how many product units “**x**” should be sold per sales

period to *optimize* this product’s PROFIT “**P**” and the value of “**P**” at

this “**x**” value. Assume market constraints suggest the maximum

number of product units that actually can be sold per sales period may

not exceed…

(1). …**510** (0 ≤ **x** ≤ 510 units). (2). …**300** (0 ≤ **x** ≤ 300 units).

d. ***[****4****]*** Use *differential calculus* to provide an estimate of *marginal*

PROFIT “*m***P**” for this product if initially…

(1). …**480** units were sold. (2). …**300** units were sold.

**Section C:** The Most Appropriate Model

1. ***[****4****]*** Identify which of the two PROFIT models derived above—1st or 2nd

order—is most appropriate for estimating purposes, according to the

“highest percent variation explained” criterion—a criterion based on

[(*R2*)] or [(*R2*)adj]. Based on which of the two models you feel is most

appropriate, would you say that the results for the 1st order or 2nd order

model are most realistic?

**PART II**

As manager of product line Blue03, you have the following data available for the past **6** sales periods. This data associates your product line’s demand (units sold) “**x**” and unit price “**p**” results for these sales periods.

|  |  |
| --- | --- |
| Product | **Blue03** |
| Demand [**x**] | Price [**p**] |
| **200** | **800** |
| **400** | **900** |
| **600** | **500** |
| **1200** | **600** |
| **1600** | **150** |
| **2000** | **50** |

**Section A: DEMAND Model Development**

1. *1st Order:* Use theChart feature of Microsoft Excel® to help derive…

a. *[2]* …the product’s “best fitting” 1st order model **p** = ƒ(**x**).

b. *[2]* …the model’s *coefficient of determination* “[(*R2*)]”. Then,

interpret the “[(*R2*)]” value.

2. *2nd Order:* If the “[(*R2*)]” value of the 1st order model is not “**+1**”, use

theChart feature of Microsoft Excel® to help derive…

a. *[2]* …the “best fitting” *polynomial*, 2nd order model **p** = ƒ(**x**).

b. *[3]* …identify the model’s *coefficient of determination* “[(*R2*)]”, and

compute its “adjusted” *coefficient of determination* “[(*R2*)adj]”. Then,

interpret the “[(*R2*)adj]” value.

**Section B: Developing the Models to be Used in Subsequent Analyses**

1. *[3]* **DEMAND**. Identify which of the DEMAND models derived

above—1st order or 2nd order—best meets our course’s “highest percent

variation explained” criterion. Use this model to answer the questions

that follow.

2. *[3]* **REVENUE**. Create the REVENUE model **R** = *f*(**x**) from the

DEMAND model identified in “1” above.

3. *[3]* **COST**, **REVENUE** and **PROFIT**. Assume you had comparable

COST “**C**” and units produced “**x**” data for the same 6 sales periods, and,

after using Excel’s Chart feature to develop 1st and 2nd order “trend line”

equations and appropriate “[(*R2*)]” values, you selected the 2nd order

equation **C** = –0.1515(**x**)2 + 345.01(**x**) + 137559 to use in further

analyses. Create the PROFIT model **P** = *f*(**x**) from the COST model and

from the REVENUE model identified in “2” above.

**Section C: “Break Even”, Optimization and Advanced Topics** [0 ≤ **x** ≤ 1,100 units]

1. *[3]* Calculate how many product units “**x**” must be produced and sold in

order to generate a PROFIT of $**0.00**. Assume market constraints are

currently such that “**x**” cannot exceed **1,100** units per sales period.

2. *[4]* Determine “**C**” and “**R**” at the quantity “**x**” where “**P**” = $**0.00**.

3. *Differential calculus* may be used as part of a process to develop

optimization estimates for “**R**max” and “**P**max”. Based on the market

constraints shown below, calculate the number of product units “**x**” that

should be sold per sales period to maximize REVENUE and PROFIT

…then…calculate “**R**max” and “**P**max” at these “**x**” values.

a. *[4]* **1,100** units (0 ≤ **x** ≤ 1,100). b. *[4]* …**850** units (0 ≤ **x** ≤ 850).

4. Determine the unit price “**p**” that should be charged per sales period to

*optimize* this product’s “**R**” and “**P**” based on the constraints of…

a. *[4]* …3a above (0 ≤ **x** ≤ 1,100 units). b. *[4]* …3b above (0 ≤ **x** ≤ 850 units).

5. *[5]* Using your product line’s **Cost**, **Revenue** and **Profit** models derived

earlier, verify the following principle from economics: at the value of

“**x**” (units produced and sold) where **Profit** “**P**” is a maximum, *marginal*

**Cost** “*m***C**” = *marginal* **Revenue** “*m***R**”.

6. Using *differential calculus* where necessary…

a. *[3]* …find the value of the independent variable “**x**” associated with

maximum *average* PROFIT “*a***P**max” for this product line.

b. *[3]* …develop the product line’s *marginal* PROFIT [“*m***P**” or (**P**)′]

expression.

c. *[3]* …verify the assertion from econometrics that at the value of “**x**”

associated with a product line’s “*a***P**max”, *average* PROFIT and

*marginal* PROFIT for this product line are equal.

**Extra Credit (**optional**)**

***EC1***. Corporate headquarters originally set your product line’s PROFIT expectation for the next sales period at $**200,000**. Is this PROFIT expectation realistic? Support your answer quantitatively and/or graphically.

***EC2***. The “most appropriate” demand equation for a particular product is found to be **x** = 2,000 – 0.625(**p**). Develop this product’s coefficient of *elasticity* expression and its Revenue equation **R** = *f*(**p**). Then, assuming there are no severe *domain* restrictions on price, determine the price where maximum Revenue occurs and the price associated with unit *elasticity*

(**ɳ** = –1). What do you observe about the two values?

[PS1(**Red03Blue03**)2011Jul]